

YSU Department of Mathematics and Mechanics
YSU - ISTC Joint Master Program

Applied Statistics and Data Science
Entrance Exam Sample Test

Exam Time: 2 hours

Last Name: _____ First Name: _____

READ THESE INSTRUCTIONS CAREFULLY

- This test consists of 10 Show-Work Problems.
- Each Show-Work Problem will be graded 10 points.
- This is a closed-book test, and no notes, assignments, practice problems, books, formula sheets or other materials are allowed.
- The use of mobile phones or any other electronic devices are strongly prohibited. Only ordinary calculators are allowed. Please turn off your cell phones and place them out of reach.
- Talking to another student, looking at another student's paper, or communicating with other students in any way is strictly forbidden.
- Use the scratch pages of the test booklet to do your draft calculations. Please ask proctors for extra scratch papers if necessary.
- If you run out of the space on the test pages, please use a scratch page to finish your work. Indicate in the test page that you will continue on the scratch page, and mark with the rectangle the portion on the scratch page that contains the solution. Any other work on the scratch page will not be graded.
- Good luck!

DO NOT OPEN THIS BOOKLET
UNTIL YOU HAVE BEEN TOLD TO DO SO

Scratch Paper

Show-Your-Work Problems

1. Plot the graph of

$$f(x) = \frac{1}{2} \cdot (|x + 1| + 0.8x), \quad x \in \mathbb{R}.$$

Scratch Paper

2. Assume $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a twice differentiable real-valued function and

$$g(x_1, x_2) = f(w_1 \cdot x_1 + w_2 \cdot x_2 + b_1, w_2 \cdot x_1 + w_1 \cdot x_2 + b_2), \quad x_1, x_2 \in \mathbb{R}$$

where w_1, w_2, b_1, b_2 are some given, fixed constants.

- a. Calculate the first order partial derivatives of g and find its gradient;
- b. Calculate the second order partial derivatives $\frac{\partial^2 g}{\partial x_1^2}, \frac{\partial^2 g}{\partial x_1 \partial x_2}$ of g .

Scratch Paper

3. Find the stationary points of

$$f(x_1, x_2) = -x_1^2 + x_2^4 - 2x_1 - 8x_2^2 - 3.$$

Supplementary, non-graded Check if the obtained points are local (global) minimum/maximum points of f .

Scratch Paper

4. Calculate the area under the graph of

$$f(x) = x \sin x^2, \quad x \in [0, \sqrt{\pi}].$$

Scratch Paper

5. Let A, B, C be 3×3 matrices with $\det A = 2, \det B = 3, \det C = 4$. Find the determinant $\det(AB^2C^TB^{-1})$.

Scratch Paper

6. Find the value of d for which the plane $z + 2y - 3x = d$ passes through the point $(1, 2, 3)$. Check that it also passes through $(0, 2, 0)$ and $(-1, 1, -1)$. Is the point $(2, 3, 5)$ above or below the plane $z + 2y - 3x = d$?

Scratch Paper

7. Our game is the following: we are given 10 boxes with 1 ball, 15 boxes with 2 balls, 20 boxes with 3 balls, and 5 boxes with no ball. We open a box, if the number of balls is odd, we get 5 points. Otherwise, we loose 2 points. What is the expected value of the points one can get in a game?

Scratch Paper

8. Assume X is a r.v. (random variable) with the PDF (probability density function)

$$f(x) = \begin{cases} \frac{5x^4}{32}, & x \in [-2, 0]; \\ 0, & \text{otherwise.} \end{cases}$$

- a. Calculate the probability $\mathbb{P}(-1 \leq X \leq 1)$;
- b. Calculate $F(-1)$, where F is the CDF (Cumulative Distribution Function) of X ;
- c. Calculate the expected value $\mathbb{E}(X)$ and the variance $\text{Var}(X)$;
- d. Which event is more probable - $X \in [-0.001, 0]$ or $X \in [-1.001, -1]$?

Scratch Paper

9. What will print the following program?

```
S = 0

for i = 1 to 10
{
  if (i is even) {
    S = S*x+i;
    {

  else {
    S = S*x -i;
    }
}

print S
```

Scratch Paper

10. Along the street on which I live, trams run in red and blue colors, running on the same route. The number of both trams is the same. Red trams, as well as blue trams, go with the same time interval of ten minutes. During the day I make several trips, and at very different times. It would seem that the number of trips in trams of red and blue should be approximately the same with a possible slight deviation. However, due to some circumstances, the actual number of trips in trams of red color is almost 90% of the number of all trips. How can such a phenomenon be explained?

Scratch Paper

Scratch Paper