

YSU Department of Mathematics and Mechanics  
YSU - ISTC Joint Master Program

Applied Statistics and Data Science  
Entrance Exam Sample Test

*Exam Time: 2 hours*

Last Name: \_\_\_\_\_ First Name: \_\_\_\_\_

READ THESE INSTRUCTIONS CAREFULLY

- This test consists of 10 Show-Work Problems.
- Each Show-Work Problem will be graded 10 points.
- This is a closed-book test, and no notes, assignments, practice problems, books, formula sheets or other materials are allowed.
- The use of mobile phones or any other electronic devices are strongly prohibited. Only ordinary calculators are allowed. Please turn off your cell phones and place them out of reach.
- Talking to another student, looking at another student's paper, or communicating with other students in any way is strictly forbidden.
- Use the scratch pages of the test booklet to do your draft calculations. Please ask proctors for extra scratch papers if necessary.
- If you run out of the space on the test pages, please use a scratch page to finish your work. Indicate in the test page that you will continue on the scratch page, and mark with the rectangle the portion on the scratch page that contains the solution. Any other work on the scratch page will not be graded.
- Good luck!

DO NOT OPEN THIS BOOKLET  
UNTIL YOU HAVE BEEN TOLD TO DO SO

# Scratch Paper

## Show-Your-Work Problems

1. Plot the graph of

$$f(x) = \frac{1}{3} \cdot \sin(2x + 1), \quad x \in \mathbb{R}.$$

# Scratch Paper

2. Find all stationary points of

$$f(x_1, x_2) = 4x_1^2 + x_2^2 - 4x_1 + 14x_2 - 31.$$

**Supplementary, non-graded** Check if the obtained points are local (global) minimum/maximum points of  $f$ .

# Scratch Paper

3. Calculate the area under the graph of

$$f(x) = \frac{x}{e^x}, \quad x \in [0, 4].$$

# Scratch Paper



4. Let

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We define

$$f(\mathbf{x}) = \mathbf{x}^T \cdot A \cdot \mathbf{x}.$$

- a. Write  $f(\mathbf{x}) = f(x_1, x_2)$  in the expanded form, as a function of  $x_1$  and  $x_2$ ;
- b. Calculate the gradient of  $f$ .

## Scratch Paper

5. For  $\mathbf{x} = [x_1, x_2, x_3]^T \in \mathbb{R}^3$  we define

$$\|\mathbf{x}\|_1 = |x_1| + |x_2| + |x_3|.$$

Prove that for  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ ,

$$\|\mathbf{x} + \mathbf{y}\|_1 \leq \|\mathbf{x}\|_1 + \|\mathbf{y}\|_1.$$

## Scratch Paper

6. Show that the line  $y - 4x + 1 = 0$  passes through the points  $(1, 3)$  and  $(3, 11)$ . Show on the Cartesian plane the set of all points satisfying  $y - 4x + 1 \geq 0$ .

## Scratch Paper

7. Our game is the following: we are rolling 2 fair dice. If 2 numbers shown on dice are equal, we get 10 points. If these numbers differ by 1, then we get 3 points. Otherwise, we loose 3 points. What is the expected value of the points one can get in a game?

## Scratch Paper



8. Assume  $X$  is a r.v. (random variable) with the PDF (Probability Density Function)

$$f(x) = \begin{cases} \frac{3x^2}{8}, & x \in [0, 2]; \\ 0, & \text{otherwise.} \end{cases}$$

- a. Calculate the probability  $\mathbb{P}(1 < X \leq 3)$ ;
- b. Calculate  $F(2)$ , where  $F$  is the CDF (Cumulative Distribution Function) of  $X$ ;
- c. Calculate the expected value  $\mathbb{E}(X)$  and the variance  $\text{Var}(X)$ ;
- d. Which event is more probable:  $X \in [0, 0.001]$  or  $X \in [1, 1.001]$ ?

## Scratch Paper

9. What will print the following program?

```
S = 0

for i = 1 to 10
{
  if (the remainder of i when dividing to 3 is not 0) {
    S = S+i;
  }

  else {
    S = S + 3*i;
  }
}

print S
```

**Supplementary, non-graded:** What if we will take 1000 instead of 10 in the *for* loop?

## Scratch Paper

10. There is a round deep lake with a diameter of 200 meters and two trees, one of which grows on the shore near the water itself, the other - in the center of the lake on a small island. A person who does not know how to swim needs to move to the island with a rope, which is a little over 200 meters in length. How can he do this?

## Scratch Paper

## Scratch Paper